Problem 1.2

Two vectors are given as $\mathbf{b} = (1, 2, 3)$ and $\mathbf{c} = (3, 2, 1)$. (Remember that these statements are just a compact way of giving you the components of the vectors.) Find $\mathbf{b} + \mathbf{c}$, $5\mathbf{b} - 2\mathbf{c}$, $\mathbf{b} \cdot \mathbf{c}$, and $\mathbf{b} \times \mathbf{c}$.

Solution

Write the given vectors as follows.

$$\mathbf{b} = \langle 1, 2, 3 \rangle$$
$$\mathbf{c} = \langle 3, 2, 1 \rangle$$

Now make the calculations.

$$\begin{aligned}
\mathbf{b} + \mathbf{c} &= \langle 1, 2, 3 \rangle + \langle 3, 2, 1 \rangle \\
&= \langle 1 + 3, 2 + 2, 3 + 1 \rangle \\
&= \langle 4, 4, 4 \rangle
\end{aligned}$$

$$\begin{aligned}
5\mathbf{b} - 2\mathbf{c} &= 5\langle 1, 2, 3 \rangle - 2\langle 3, 2, 1 \rangle \\
&= \langle 5, 10, 15 \rangle - \langle 6, 4, 2 \rangle \\
&= \langle 5 - 6, 10 - 4, 15 - 2 \rangle \\
&= \langle -1, 6, 13 \rangle
\end{aligned}$$

$$\begin{aligned}
\mathbf{b} \cdot \mathbf{c} &= \langle 1, 2, 3 \rangle \cdot \langle 3, 2, 1 \rangle \\
&= (1)(3) + (2)(2) + (3)(1) \\
&= 10
\end{aligned}$$

$$\begin{aligned}
\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} \hat{\mathbf{x}} - \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} \hat{\mathbf{y}} + \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \hat{\mathbf{z}} \\
&= [(2)(1) - (3)(2)]\hat{\mathbf{x}} - [(1)(1) - (3)(3)]\hat{\mathbf{y}} + [(1)(2) - (2)(3)]\hat{\mathbf{z}} \\
&= -4\hat{\mathbf{x}} + 8\hat{\mathbf{y}} - 4\hat{\mathbf{z}} \\
&= \langle -4, 8, -4 \rangle
\end{aligned}$$